

# Updated SAID analysis of pion photoproduction data

Ron L. Workman, William J. Briscoe, Mark W. Paris, Igor I. Strakovsky<sup>1</sup>

<sup>1</sup>*Institute for Nuclear Studies, Department of Physics,  
The George Washington University, Washington, D.C. 20052*

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Energy-dependent and single-energy fits to the existing pion photoproduction database have been updated to cover the region from threshold to 2.7 GeV in the laboratory photon energy. Revised resonance photo-decay couplings have been extracted and compared to previous determinations. The influence of recent measurements is displayed. Remaining problems and future approaches are discussed.

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## I. INTRODUCTION

The SAID photoproduction analyses have been updated periodically since 1990 [1], with more frequent updates published through our website [2]. Our last full analysis [3] has been revised twice [4, 5] to include CLAS differential cross sections for neutral and charged pion production off proton targets. Some recent neutron target data were poorly predicted by these and the MAID [6] solutions, requiring changes in the neutron multipoles and resonance couplings. Further changes are expected with the incorporation of forthcoming data from JLab FROST [7] and HD-ICE [8], CBMAMI [9], LEPS [10], and CB-ELSA [11]. Here we compare with previous fits and consider what changes can be expected with future additions to the database and changes in the SAID parametrization.

In Section II we summarize changes to the SAID database. The changes reflected in our multipoles are displayed in Section III. A comparison of past and recent photo-decay amplitudes, for resonances giving a significant contribution to pion photoproduction, is made in Section IV. Finally, in Section V, we summarize our results and comment on possible changes due to further measurements and changes in our parametrization form.

## II. DATABASE

The most influential additions to our database have been recent measurements of the photon beam asymmetry  $\Sigma$  for  $\bar{\gamma}n \rightarrow \pi^-p$  [12] and for  $\bar{\gamma}n \rightarrow \pi^0n$  [13] from GRAAL. These include 216  $\Sigma$  measurements of  $\pi^0n$  covering  $E_\gamma=703\text{--}1475$  MeV and  $\theta=53\text{--}164^\circ$  plus 99  $\Sigma$  measurements of  $\pi^-p$  for  $E_\gamma=753\text{--}1439$  MeV and  $\theta=33\text{--}163^\circ$ .

We note that the GRAAL contribution to  $\pi^0n$  has doubled the world database for this reaction. Our best fit (SN11) for  $\pi^0n$  and  $\pi^-p$ , reduces the initial  $\chi^2/\text{data}$  from 223 and 27 (for the SAID energy-dependent solution SP09 [4]) to 3.1 and 4.6, respectively. In particular, this shows that the  $\pi^0n$  data were not well predicted, based on the existing large proton-target database and the much smaller  $\pi^-p$  dataset.

Cross section [14, 15],  $\Sigma$  [16, 17], and double-polarized  $C_{x'}$  [18] data for  $\gamma p \rightarrow \pi^0p$  have had a lesser impact. For this reason, in the next section we will focus mainly on the neutron target fits and multipoles.

## III. MULTIPOLE AMPLITUDES

The present multipole analysis retains the phenomenological form used in Ref. [3], which extended the multipole parametrization based on a Heitler  $K$ -matrix approach [1, 19],

$$M = (\text{Born} + A)(1 + iT_{\pi N}) + BT_{\pi N}, \quad (1)$$

to include a term of the form

$$(C + iD)(\text{Im}T_{\pi N} - |T_{\pi N}|^2), \quad (2)$$

where  $T_{\pi N}$  is the elastic  $\pi N$  scattering partial-wave amplitude associated with the pion-photoproduction multipole amplitude  $M$ . This new piece was found necessary to fit the increasingly precise polarization data, and has recently been used in a study of the model-independence of energy-dependent and single-energy fits [20]. The factors  $A$  through  $D$  were parametrized in terms of simple polynomials with the correct threshold behavior. Other forms, such as the Chew-Mandelstam (CM) parametrization [21],

$$M = \sum_{\sigma} [1 - \bar{K}C]_{\pi\sigma}^{-1} \bar{K}_{\sigma\gamma}, \quad (3)$$

employing CM  $K$ -matrix elements,  $\bar{K}_{\pi\sigma}$ , determined in a fit to  $\pi N$  elastic scattering data [22], have also been explored [21].

The multipole amplitudes are presented in terms of isospin states, as is the convention. Extending such an analysis below the  $\pi^+n$  threshold is clearly problematic. This region is not the focus of the present study, and requires a separate analysis. In fits after SM95 [23], Arndt proposed a recipe whereby the above  $\pi N$  partial-wave  $T$ -matrices were evaluated in terms of the outgoing pion energy of a corresponding photoproduction reaction rather than the center-of-mass energy. This method allowed

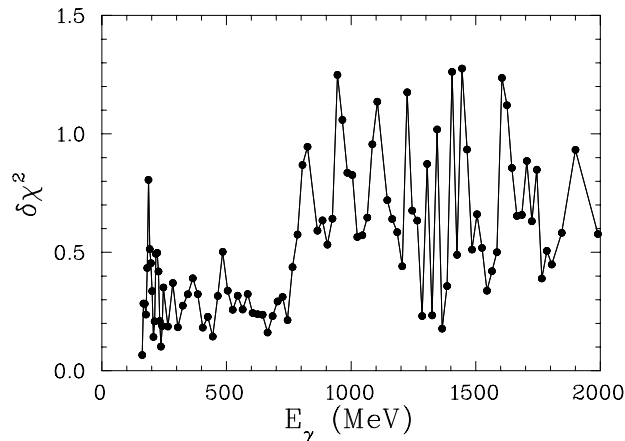


FIG. 1: Comparison of the SES and global SN11 fits via  $\delta\chi^2 = [\chi^2(\text{SN11}) - \chi^2(\text{SES})]/N_{\text{data}}$  versus lab photon energy  $E_\gamma$ .

rather good fits to the threshold data, but resulted in a charge-state-dependent shift of the  $\pi N$   $T$ -matrix pole positions, by a few MeV, depending on whether the  $\pi^0 p$  or  $\pi^+ n$  final states were being analyzed. Here we have made fits, with (SK11) and without (SN11) this kinematic shift, to gauge its influence on the photo-decay amplitudes. The fit quality, in terms of chi-squared, for these and previous SAID solutions, is compared in Table I.

TABLE I:  $\chi^2$  comparison of fits to pion photoproduction data. Results are shown for eight different SAID solutions (current SN11 and SK11 with previous SP09 [4], FA06 [5], SM02 [3], and SM95 [23]). See text for details.

Solution	Energy limit (MeV)	$\chi^2/N_{\text{Data}}$	$N_{\text{Data}}$
SN11	2700	2.08	25553
SK11	2700	2.09	25961
SP09	2700	2.11	25639
SM02	2000	2.01	17571
SM95	2000	2.37	13415

The SK11 fit extends down to the  $\pi^0 p$  threshold, at a photon energy of  $E_\gamma=144.7$  MeV, while SN11 is limited to 155 MeV, just above the  $\pi^+ n$  threshold (151.4 MeV), thus avoiding complications from the region between the  $\pi^0 p$  and  $\pi^+ n$  thresholds. The quality of the overall data fit, as shown in Table I, is nearly identical.

We have generated the single-energy solutions (SES), described extensively in Ref. [20], based on the global fit SN11. The quantity  $\delta\chi^2$  is the difference,  $[\chi^2(\text{SN11}) - \chi^2(\text{SES})]$ , divided by the number of data in each single-energy bin, providing a measure of the agreement between an individual SES and the global SN11 results (see Fig. 1). We emphasize that the SES are generated mainly to search for missing structures in the global fit. Detailed comparisons of the global and SES fits can be made on

the SAID website [2].

In Fig. 2, we display the most significant deviations of the SN11 solution from the fit SP09, published in Ref. [4], and the Mainz MAID07 [6] result for selected neutron multipoles.

The differences between our SN11 and SP09 results for neutron targets are visible particularly for the  $E_{0+}^{1/2}$  (Fig. 2) above  $W \approx 1280$  MeV ( $E_\gamma=400$  MeV). The  $E_{3-}^{1/2}$  multipole, connected to the  $N(1680)F_{15}$  resonance, is also quite different. ( $N(1680)$  being the PDG notation [28] and  $L_{2I,2J}$  being the associated notation for a state in  $\pi N$  elastic scattering [22].) The MAID07 fit was also modified [13] to accommodate the new  $\pi^0 n$   $\Sigma$  data, resulting in changes beginning at a higher energy, mainly altering the  $N(1720)P_{13}$  resonance parameters. Fits to these neutron target data are displayed in Figs. 3 and 4.

In Figs. 5 and 6, we show discrepancies corresponding to the double-polarization quantity ( $G$ ), at low energy, and others for the unpolarized cross section, at higher energies. In Fig. 6, we compare a prediction from SP09 to a fit including the data of Ref. [15]. A large discrepancy exists in the forward direction. Eliminating the existing CLAS cross sections [5], which do not extend to very forward angles, allows a slightly improved fit, but clearly does not resolve this problem, which exists also for the Bonn-Gatchina fits [26].

#### IV. RESONANCE COUPLINGS

In order to make meaningful comparisons [27] with previous resonance determinations, we have retained the method used in Refs. [3–5], fitting the resultant multipoles with a background plus resonance assumption, similar to that used in the MAID analysis,

$$A(W)(1 + iT_{\pi N}) + T_{\text{BW}}e^{i\phi}, \quad (4)$$

wherein  $T_{\pi N}$  is again the corresponding  $\pi N$   $T$ -matrix and  $T_{\text{BW}}$  is a Breit-Wigner (BW) parametrization of

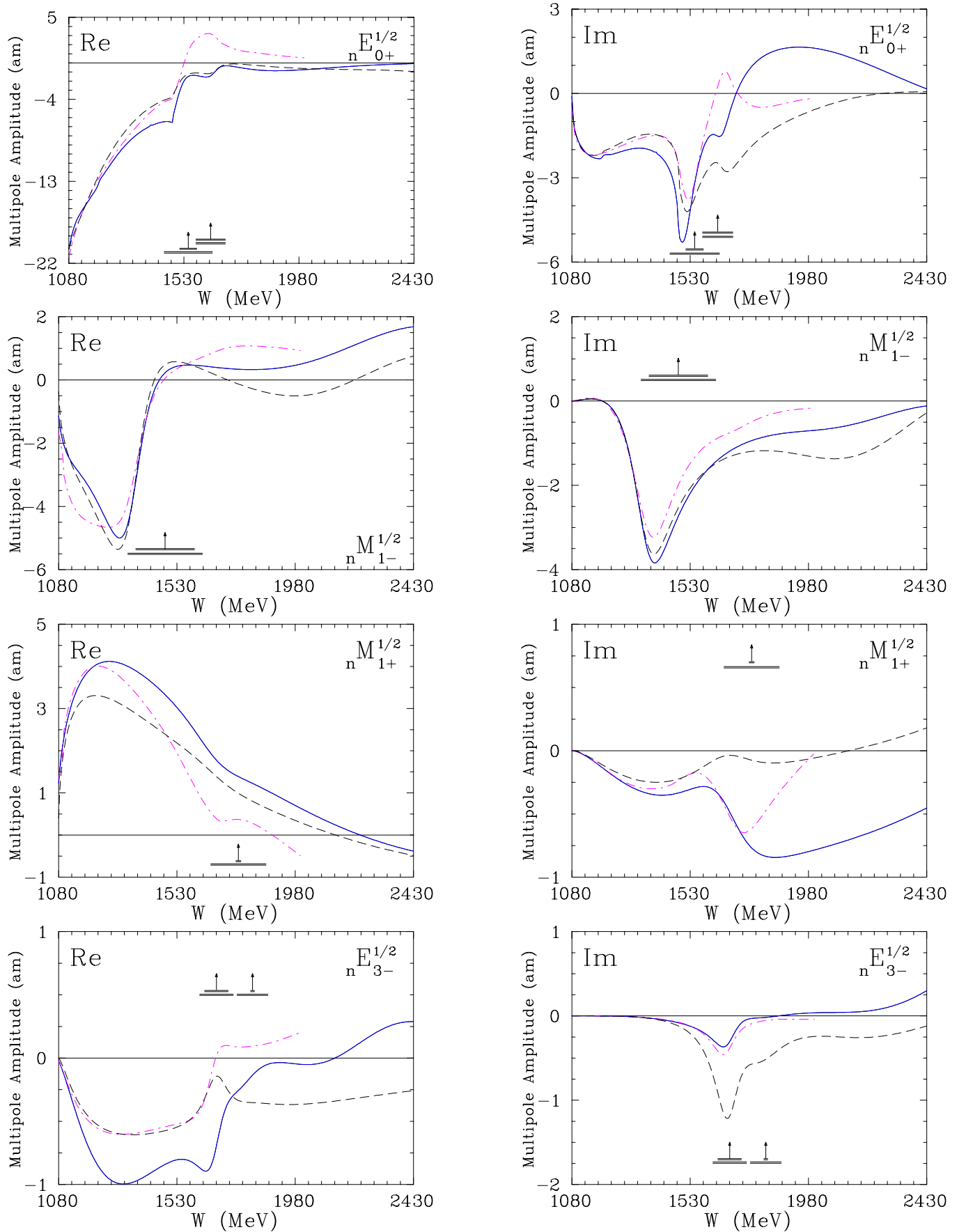


FIG. 2: (Color online) Selected neutron multipole amplitudes from threshold to  $W = 2.43$  GeV ( $E_\gamma = 2.7$  GeV). Solid lines correspond to the SN11 solution. Dashed (dash-dotted) lines give solution SP09 [4] (MAID07 [6], which terminates at  $W=2$  GeV). Vertical arrows indicate  $W_R$  and horizontal bars show full ( $\Gamma$ ) and partial ( $\Gamma_{\pi N}$ ) widths associated with the SAID  $\pi N$  solution SP06 [22].

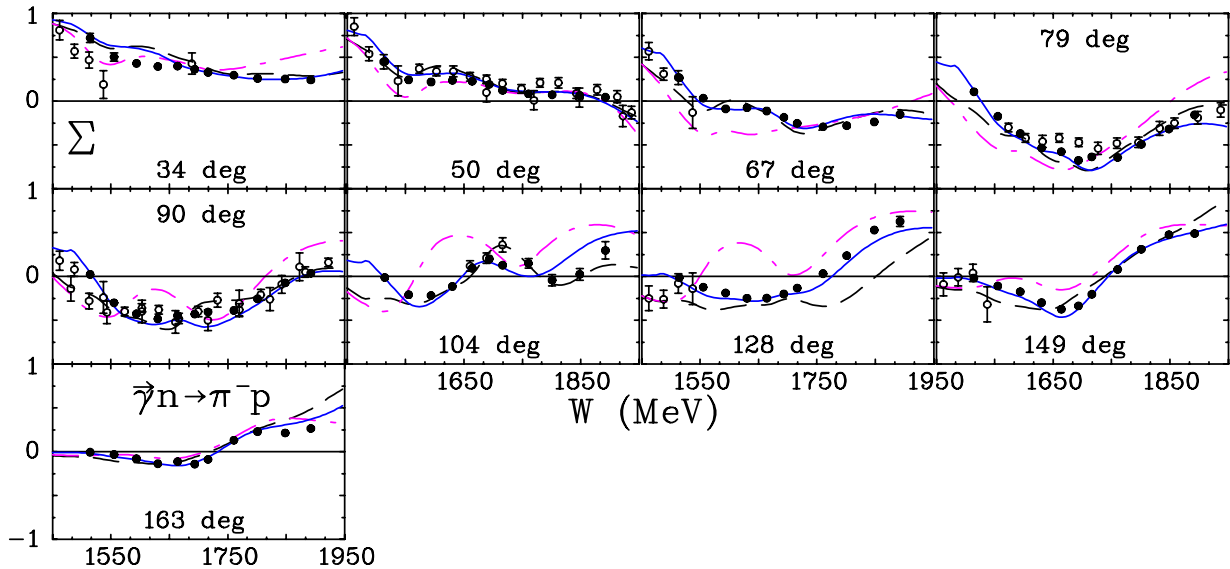


FIG. 3: (Color online)  $\Sigma$  asymmetry for  $\bar{\gamma}n \rightarrow \pi^-p$ . Data (filled circles) are from GRAAL [12]. The previous measurements (open circles) are available in the SAID database [2]. Notation of the solutions is the same as in Fig. 2. GRAAL measurements are not included in SP09 and MAID07.

the resonance contribution.  $A(W)$  is a linear approximation to the energy dependence of the Born plus phenomenological term multiplying  $(1 + iT_{\pi N})$  in Eq. (1). Here the (theoretical) systematic error in our determination is generally much larger than the statistical errors found in fitting the data over an energy bin, around the BW resonance energy, or fitting (with subjective errors) the energy-dependent or single-energy multipoles covering the same energy range. The errors quoted in Tables II and III were found by varying the energy range of the fit between the estimated resonance full and half-widths previously determined from our  $\pi N$  elastic scattering analysis [22].

The amplitudes  $A_{1/2}$  and  $A_{3/2}$  were determined assuming the masses, widths, and  $\pi N$  branching fractions determined in an earlier BW analysis of  $\pi N$  elastic scattering data [22]. In a few cases, these input parameters from  $\pi N$  scattering did not produce good fits to the photoproduction multipoles.

For the  $N(1650)S_{11}$ , increasing the mass to the nominal value of 1650 MeV produced a much better fit, as did increasing the width. The photo-decay couplings (proton and neutron) changed substantially and, as a result, this variability was taken to determine the quoted errors. The  $N(1535)S_{11}$  decay to  $p\gamma$  has remained quite

stable while the decay to  $n\gamma$  has changed significantly, due mainly to the new  $\Sigma$  data for both the  $\pi^-p$  and  $\pi^0n$  channels, as shown in terms of the multipoles and data fits in Figs. 2–4.

The  $N(1720)P_{13}$  neutron couplings are poorly determined, but the SN11 solution has, for the neutron  $M_{1+}^{1/2}$  multipole, an imaginary part now more closely resembling the MAID07 value at the BW resonance energy. The MAID07 values for the neutron  $A_{1/2}$  and  $A_{3/2}$  amplitudes are  $-3$  and  $-31 \text{ GeV}^{-1/2} \times 10^{-3}$ , in better agreement with SAID, compared to the last published values from the SM95 fit. The  $\Delta(1620)S_{31}$  amplitude is now significantly larger, and outside of the PDG estimate, but is consistent with the MAID07 [6] and Bonn-Gatchina [26] results ( $66$  and  $63 \pm 12 \text{ GeV}^{-1/2} \times 10^{-3}$ , respectively).

## V. SUMMARY AND CONCLUSION

This updated analysis examined mainly the effect of new neutron-target data on the SAID multipoles and resonance parameters. In some cases, the changes have been significant. The neutron multipoles generally show much larger variations than the proton multipoles, when the fits of different groups are compared. Given the inability

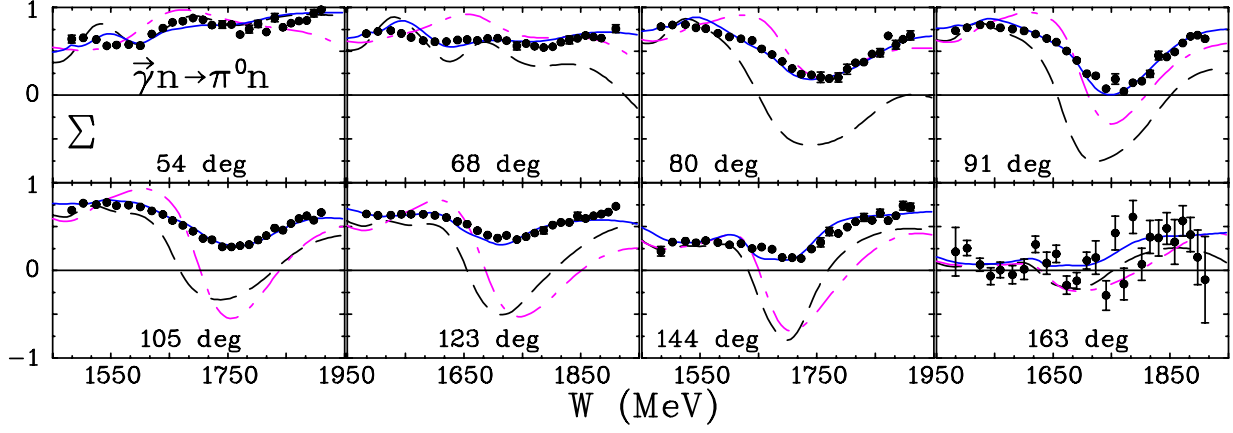


FIG. 4: (Color online)  $\Sigma$  asymmetry for  $\bar{\gamma}n \rightarrow \pi^0 n$ . Data (filled circles) are from GRAAL [13]. Notation of the solutions is the same as in Fig. 2. GRAAL measurements not included in SP09 and MAID07.

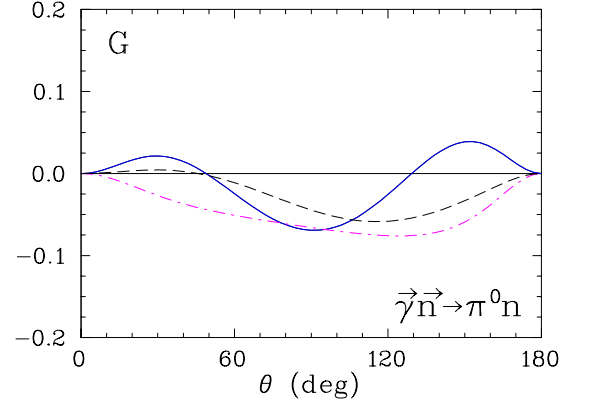
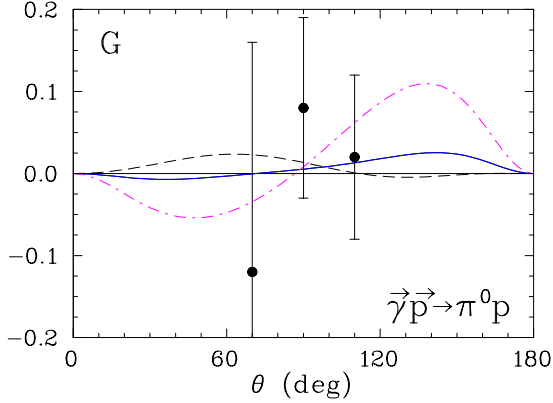


FIG. 5: (Color online)  $G$  asymmetry for neutral-pion photoproduction in the  $\Delta$  resonance region ( $E_\gamma=340$  MeV). Data (filled circles) are from MAMI [24]. Notation of the solutions is the same as in Fig. 2.

of fits, based mainly on proton target data, to predict the  $\pi^0 n$  multipoles, further changes can be expected as more neutron-target data become available.

Apart from a few special cases, the photo-decay amplitudes,  $A_{1/2}$  and  $A_{3/2}$ , found in this study are reasonably consistent with the PDG averages. The  $N(1535)S_{11}$  and  $N(1650)S_{11}$  amplitudes deserve further discussion, though they are now also consistent with the PDG estimates. The large PDG uncertainty, assigned to the  $N(1535)S_{11}$ , was due mainly to a disagreement which

existed between values determined from eta photoproduction fits, and existing values from pion photoproduction. Roughly, in 1995, the eta photoproduction value for  $A_{1/2}$  [29] was twice the SAID SM95 value from pion photoproduction. While the SAID value has migrated up to a value consistent with the early eta photoproduction estimates, the MAID determination has decreased, once again leaving a wide discrepancy. The increased SAID value for the  $N(1650)S_{11}$  amplitude appears to be due more to the extraction technique than any change in the

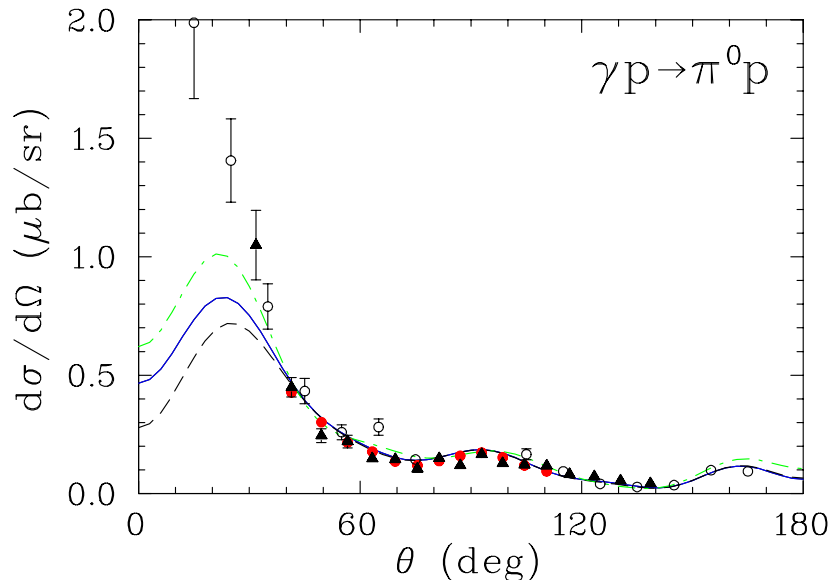


FIG. 6: (Color online) Differential cross section for  $\gamma p \rightarrow \pi^0 p$  at  $E_\gamma = 2225$  MeV. CLAS data (filled circles) are from [5]. CB-ELSA data (open circles [15] and filled triangles [25]). Notation of the solutions is the same as in Fig. 2. SZ11 solution (CLAS  $\pi^0 p$  cross sections [4] excluded) is shown by a dash-dotted line.

multipole. As we mentioned above, this extraction was very sensitive to assumed values for the mass and width, which may have produced the low value in Ref. [5].

The use of Eq. (4) in extracting the above values of  $A_{1/2}$  and  $A_{3/2}$  is reasonably consistent with the MAID approach and earlier extractions by Berends and Donachie [30] and by Crawford and Morton [31]. It may not, however, be consistent with determinations [26] based on the  $K$ - or  $T$ -matrix pole. This complicates comparisons, particularly for multipoles without clear resonance signatures. We hope to address this point in future studies.

Finally, we mention that preliminary fits to photoproduction data, using the CM formalism of Eq. (3), discussed in Ref. [21], are qualitatively similar to, but

quantitatively different from, the results presented here. This form, which uses a more constrained approach to the incorporation of higher opening channels ( $\eta N$ ,  $\pi \Delta$ ,  $\rho N$ ), essentially replaces the behavior of the term given in Eq. (2) (proportional to the reaction cross section) by terms contributing to each channel separately. A more detailed comparison is in progress.

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TABLE II: Resonance parameters for  $N^*$  and  $\Delta^*$  states from the SAID fit to the  $\pi N$  data [22] (second column) and proton helicity amplitudes  $A_{1/2}$  and  $A_{3/2}$  (in  $[(\text{GeV})^{-1/2} \times 10^{-3}]$  units) from the SN11 solution (first row), previous SP09 [4] solution (second row), and average values from the PDG10 [28] (third row).

Resonance	$\pi N$ SAID	$A_{1/2}$	$A_{3/2}$
$N(1535)S_{11}$	$W_R=1547$ MeV	$99 \pm 2$	
	$\Gamma=188$ MeV	$100.9 \pm 3.0$	
	$\Gamma_\pi/\Gamma=0.36$	$90 \pm 30$	
$N(1650)S_{11}$	$W_R=1635$ MeV	$65 \pm 25$	
	$\Gamma=115$ MeV	$9.0 \pm 9.1$	
	$\Gamma_\pi/\Gamma=1.00$	$53 \pm 16$	
$N(1440)P_{11}$	$W_R=1485$ MeV	$-58 \pm 1$	
	$\Gamma=284$ MeV	$-56.4 \pm 1.7$	
	$\Gamma_\pi/\Gamma=0.79$	$-65 \pm 4$	
$N(1720)P_{13}$	$W_R=1764$ MeV	$99 \pm 3$	$-43 \pm 2$
	$\Gamma=210$ MeV	$90.5 \pm 3.3$	$-36.0 \pm 3.9$
	$\Gamma_\pi/\Gamma=0.09$	$18 \pm 30$	$-19 \pm 20$
$N(1520)D_{13}$	$W_R=1515$ MeV	$-16 \pm 2$	$156 \pm 2$
	$\Gamma=104$ MeV	$-26.0 \pm 1.5$	$141.2 \pm 1.7$
	$\Gamma_\pi/\Gamma=0.63$	$-24 \pm 9$	$166 \pm 5$
$N(1675)D_{15}$	$W_R=1674$ MeV	$13 \pm 2$	$19 \pm 2$
	$\Gamma=147$ MeV	$14.9 \pm 2.1$	$18.4 \pm 2.1$
	$\Gamma_\pi/\Gamma=0.39$	$19 \pm 8$	$15 \pm 9$
$N(1680)F_{15}$	$W_R=1680$ MeV	$-13 \pm 3$	$141 \pm 3$
	$\Gamma=128$ MeV	$-17.6 \pm 1.5$	$134.2 \pm 1.6$
	$\Gamma_\pi/\Gamma=0.70$	$-15 \pm 6$	$133 \pm 12$
$\Delta(1620)S_{31}$	$W_R=1615$ MeV	$64 \pm 2$	
	$\Gamma=147$ MeV	$47.2 \pm 2.3$	
	$\Gamma_\pi/\Gamma=0.32$	$27 \pm 11$	
$\Delta(1232)P_{33}$	$W_R=1233$ MeV	$-138 \pm 3$	$-259 \pm 5$
	$\Gamma=119$ MeV	$-139.6 \pm 1.8$	$-258.9 \pm 2.3$
	$\Gamma_\pi/\Gamma=1.00$	$-135 \pm 6$	$-250 \pm 8$
$\Delta(1700)D_{33}$	$W_R=1695$ MeV	$109 \pm 4$	$84 \pm 2$
	$\Gamma=376$ MeV	$118.3 \pm 3.3$	$110.0 \pm 3.5$
	$\Gamma_\pi/\Gamma=0.16$	$104 \pm 15$	$85 \pm 22$
$\Delta(1905)F_{35}$	$W_R=1858$ MeV	$9 \pm 3$	$-46 \pm 3$
	$\Gamma=321$ MeV	$11.4 \pm 8.0$	$-51.0 \pm 8.0$
	$\Gamma_\pi/\Gamma=0.12$	$26 \pm 11$	$-45 \pm 20$
$\Delta(1950)F_{37}$	$W_R=1921$ MeV	$-71 \pm 2$	$-92 \pm 2$
	$\Gamma=271$ MeV	$-71.5 \pm 1.8$	$-94.7 \pm 1.8$
	$\Gamma_\pi/\Gamma=0.47$	$-76 \pm 12$	$-97 \pm 10$



TABLE III: Resonance parameters for  $N^*$  states from the SAID fit to the  $\pi N$  data [22] (second column) and neutron helicity amplitudes  $A_{1/2}$  and  $A_{3/2}$  (in  $[(\text{GeV})^{-1/2} \times 10^{-3}]$  units) from the SN11 solution (first row), previous SM02 [3] solution (second row), and average values from the PDG10 [28] (third row). <sup>†</sup>SM95 value [23]

Resonance	$\pi N$ SAID	$A_{1/2}$	$A_{3/2}$
$N(1535)S_{11}$	$W_R=1547$ MeV $\Gamma=188$ MeV $\Gamma_\pi/\Gamma=0.36$	$-60\pm3$ $-16\pm5$ $-46\pm27$	
$N(1650)S_{11}$	$W_R=1635$ MeV $\Gamma=115$ MeV $\Gamma_\pi/\Gamma=1.00$	$-26\pm8$ $-28\pm4$ $-15\pm21$	
$N(1440)P_{11}$	$W_R=1485$ MeV $\Gamma=284$ MeV $\Gamma_\pi/\Gamma=0.79$	$48\pm4$ $45\pm15$ $40\pm10$	
$N(1720)P_{13}$	$W_R=1764$ MeV $\Gamma=210$ MeV $\Gamma_\pi/\Gamma=0.09$	$-21\pm4$ $7\pm15^\dagger$ $1\pm15$	$-38\pm7$ $-5\pm25^\dagger$ $-29\pm61$
$N(1520)D_{13}$	$W_R=1515$ MeV $\Gamma=104$ MeV $\Gamma_\pi/\Gamma=0.63$	$-47\pm2$ $-67\pm4$ $-59\pm9$	$-125\pm2$ $-112\pm3$ $-139\pm11$
$N(1675)D_{15}$	$W_R=1674$ MeV $\Gamma=147$ MeV $\Gamma_\pi/\Gamma=0.39$	$-42\pm2$ $-50\pm4$ $-43\pm12$	$-60\pm2$ $-71\pm5$ $-58\pm13$
$N(1680)F_{15}$	$W_R=1680$ MeV $\Gamma=128$ MeV $\Gamma_\pi/\Gamma=0.70$	$50\pm4$ $29\pm6$ $29\pm10$	$-47\pm2$ $-58\pm9$ $-33\pm9$